12.4 Volume of Prisms, Cylinders, Pyramids, and Cones

Geometry
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Spring 2013
Geometry Bell Ringer

Find the volume of the cylinder with a radius of 7 in. and a height of 10 in. Please leave your answer in terms of $\pi$.

- a. $140\pi$ in.$^3$
- b. $490\pi$ in.$^3$
- c. $70\pi$ in.$^3$
- d. $245\pi$ in.$^3$
Geometry Bell Ringer

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d. \( 245\pi \text{ in.}^3 \)

Answer: B
Daily Learning Target (DLT) Thursday January 31, 2013

• “I can understand, apply, and remember how to find the volume of prisms, cylinders, pyramids, and cones.”
Upcoming Events:

- Sample EOC Tests May Be Taken Next Week
- Juniors – ACT Testing March 5, 2012
- There’s an ACT Math Practice Test Link On Your Class Website
- I also have ACT Practice Books If Interested
11.4 Assignment
Pages 627-630 (1-19 Odds, 28, 35, 41-44)
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1. $216 \text{ ft}^3$
2. $17. h = \frac{28}{9} \text{ cm}$
3. $180 \text{ m}^3$
4. $19. h = 6 \text{ ft}$
5. $280.6 \text{ cm}^3$
6. $28. A$
7. $720 \text{ mm}^3$
8. $35. 140.6 \text{ in}^3$
9. $288\pi \text{ in}^3$
10. $41. B$
11. $37.5\pi \text{ m}^3$
12. $42. G$
13. $117.8 \text{ m}^3$
14. $43. C$
15. $501 \text{ in}^3$
16. $44. F$
17. $3445 \text{ in}^3$
18. $501 \text{ in}^3$
Exploring Volume

• The volume of a solid is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic meters (m$^3$).
Volume Postulates

• **Volume of a cube**
  \[ V = s^3 \]

• **Volume of a prism**
  \[ V = B \times h \text{ Or } V = L \times W \times H \]

• **Volume Congruence Postulate**
  If two polyhedra are congruent, then they have the same volume.
Volume Theorems

• **Volume of a Prism**— The volume $V$ of a prism is $V = Bh$, where $B$ is the area of the base and $h$ is the height.

• **Volume of a Cylinder**— The volume $V$ of a cylinder is $V = Bh = \pi r^2 h$, where $B$ is the area of a base, $h$ is the height, and $r$ is the radius of the base.
Ex. 1: Finding the Volume of a rectangular prism

- The box shown is 5 units long, 3 units wide, and 4 units high. How many unit cubes will fit in the box? What is the volume of the box?
Ex. 1: Finding the Volume of a rectangular prism

• The base of the box is 5 units by 3 units. This means 5 \cdot 3, or 15 unit cubes, will cover the base. Three more layers of 15 cubes each can be placed on top of the lower layer to fill the box. Because the box contains 4 layers with 15 cubes in each layer, the box contains a total of 4 \cdot 15 cubes, or 60 unit cubes.
Conclusion

• Because the box is completely filled by the 60 cubes, and each cube has a volume of 1 cubic unit, it follows that the volume of the box is $60 \times 1$, or 60 cubic units.
Note:

• In example 1, the area of the base, 15 square units, multiplied by the height of 4 units, yields the volume of the box, 60 cubic units. So, the volume of the prism can be found by multiplying the area of the base by the height. This method can also be used to find the volume of a cylinder.
Ex. 2: Finding Volumes

- Find the volume of the right prism.

\[ A = \frac{1}{2} bh \]  
\[ A = \frac{1}{2} (3)(4) \]  
\[ A = 6 \text{ cm}^2 \]  
\[ V = Bh \]  
\[ V = (6)(2) \]  
\[ V = 12 \text{ cm}^3 \]

- Area of a triangle
- Substitute values
- Multiply values -- base
- Volume of a prism formula
- Substitute values
- Multiply values & solve
Ex. 2: Finding Volumes

- Find the volume of the right cylinder.

\[ A = \pi r^2 \]  \hspace{1cm} \text{Area of a circle}

\[ A = \pi 8^2 \]  \hspace{1cm} \text{Substitute values}

\[ A = 64\pi \text{ in.}^2 \]  \hspace{1cm} \text{Multiply values -- base}

\[ V = Bh \]  \hspace{1cm} \text{Volume of a prism formula}

\[ V = 64\pi(6) \]  \hspace{1cm} \text{Substitute values}

\[ V = 384\pi \text{ in.}^3 \]  \hspace{1cm} \text{Multiply values & solve}

\[ V = 1206.37 \text{ in.}^3 \]  \hspace{1cm} \text{Simplify}
Ex. 3: Using Volumes

- Use the measurements given to solve for x.
Ex. 3: Using Volumes

- Use the measurements given to solve for x.

**Solution**

a. A side length of the cube is $x$ feet.

$$V = s^3$$  \[100 = x^3\]

4.64 ≈ $x$

So, the height, width, and length of the cube are about 4.64 feet.
Ex. 3: Using Volumes

- Use the measurements given to solve for $x$. 

b. Right cylinder, $V = 4561 \text{ m}^3$
Ex. 3: Using Volumes

- Use the measurements given to solve for $x$.

b. Right cylinder, $V = 4561 \, \text{m}^3$

The area of the base is $\pi x^2$ square meters.

$$V = Bh$$

$$4561 = \pi x^2 (12)$$

$$4561 = 12\pi x^2$$

$$\frac{4561}{12\pi} = x^2$$

$$11 \approx x$$

So, the radius of the cylinder is about 11 meters.
Ex. 4: Using Volumes in Real Life

- Construction. Concrete weighs 145 pounds per cubic foot. To find the weight of the concrete block shown, you need to find its volume. The area of the base can be found as follows:

\[ B = \text{Area of large rectangle} - 2 \cdot \text{Area of small rectangle} \]

\[ = (1.31)(0.66) - 2(0.33)(0.39) \]

\[ \approx 0.61 \text{ ft}^2 \]
Ex. 4: Using Volumes in Real Life

• Using the formula for the volume of a prism, the volume is

\[ V = Bh \approx 0.61(0.66) \approx 0.40 \text{ ft.}^3 \]

\( \Rightarrow \) To find the weight of the block, multiply the pounds per cubic foot, 145 lb/ft.\(^3\), by the number of cubic feet, 0.40 ft\(^3\).

\[
\text{Weight} = \frac{145 \text{ lb}}{1 \text{ ft}^3} \cdot 0.40 \text{ ft}^3 \approx 58 \text{ lbs.}
\]
In Lesson 12.4, you learned that the volume of a prism is equal to $Bh$, where $B$ is the area of the base, and $h$ is the height. From the figure at the left, it is clear that the volume of the pyramid with the same base area $B$ and the same height $h$ must be less than the volume of the prism. The volume of the pyramid is one third the volume of the prism.
12.9—Volume of a Pyramid – The volume $V$ of a pyramid is $V = \frac{1}{3}Bh$, where $B$ is the area of the base and $h$ is the height.
12.10 Volume of a Cone – The volume of a cone is \( V = Bh = \pi r^2 h \), where \( B \) is the area of the base, \( h \) is the height and \( r \) is the radius of the base.
Ex. 1: Finding the volume of a pyramid

Find the volume of the pyramid with the regular base.
Ex. 1: Finding the volume of a pyramid

- The base can be divided into six equilateral triangles. Using the formula of an equilateral triangle, \( \frac{1}{4} \sqrt{3} \cdot s^2 \), the area of the base \( B \) can be found as follows:

\[
6 \cdot \frac{1}{4} \sqrt{3} \cdot s^2 = 6 \cdot \frac{1}{4} \sqrt{3} \cdot 3^2 = \frac{27}{2} \sqrt{3} \text{ cm}^2.
\]
Use Theorem 12.9 to find the volume of the pyramid.

**Ex. 1: Finding the volume of a pyramid**

\[
V = \frac{1}{3} Bh \\
= \frac{1}{3} \left( \frac{27}{2} \sqrt{3} \right) (4) \\
= 18\sqrt{3}
\]

So, the volume of the pyramid is \(18\sqrt{3}\), or about 31.2 cubic centimeters.
Ex. 2: Finding the volume of a cone

a. Right circular cone
Ex. 2: Finding the volume of a cone

Find the volume of each cone.

**Solution**

a. Use the formula for the volume of a cone.

\[
V = \frac{1}{3} Bh
\]

\[
= \frac{1}{3}(\pi r^2)h
\]

\[
= \frac{1}{3}(\pi 12.4^2)(17.7)
\]

\[
\approx 907.18\pi
\]

Formulas:
- **Volume of cone**
- **Base area equals \( \pi r^2 \)**
- **Substitute.**
- **Simplify.**

\[\text{So, the volume of the cone is about } 907.18\pi, \text{ or 2850 cubic millimeters.}\]
Ex. 2: Finding the volume of a cone.

Find the volume of each cone.

b. Oblique circular cone
Find the volume of each cone.

Ex. 2: Finding the volume of a cone

b. Use the formula for the volume of a cone.

\[ V = \frac{1}{3} Bh \]
\[ = \frac{1}{3} (\pi r^2)h \]
\[ = \frac{1}{3} (\pi 1.5^2)(4) \]
\[ = 3\pi \]

So, the volume of the cone is \(3\pi\), or about 9.42 cubic inches.
Ex. 3: Using the Volume of a Cone

Use the given measurements to solve for $x$.

**Solution**

\[ V = \frac{1}{3} \pi r^2 h \]

2614 = \frac{1}{3} (\pi x^2)(13)  

7842 = 13\pi x^2  

192 \approx x^2  

13.86 \approx x

So, the radius of the cone is about 13.86 feet.
11.5 Assignment
Pages 634-637 (1-17 Odds, 37-42)
Geometry Exit Quiz – 5 Points

Find the volume of the cylinder with a diameter of 6 in. and a height of 9 in. Please leave your answer in terms of $\pi$.

a. $75\pi$ in.$^3$

b. $78\pi$ in.$^3$

c. $81\pi$ in.$^3$

d. $84\pi$ in.$^3$